Diversions

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Simple Noughts and Crosses

Almost every adult in Australia has played Noughts and Crosses at some time in their life—it is that sort of game, a universal classic. However, if adults play the game, they are almost always playing with children. Very few Australian adults play Noughts and Crosses, adult against adult, unless they are bored witless and cannot think of anything better to do. It is a game they learned as children, and grew out of. Why do adults stop playing the game? Many will say it is boring. Lots will say the reason they find it boring is that they always win, because they know the winning strategy.

Challenges

Have you found a good winning strategy for Noughts and Crosses? If not, here is a problem you could try to solve: find a good winning strategy! Then, when you think you know the winning strategy for Noughts and Crosses, think again. There is *no* fool-proof winning strategy.

The main mathematical challenge of Noughts and Crosses is to find the optimum way of playing—this is, in fact, a perfect strategy for forcing a draw. If you do not know the fool-proof drawing strategy, find it.

This is typical of almost all strategy board and other games: the main mathematics challenge—problem solving—is to analyse the game and find the *best* way of playing, the optimal strategy.

First moves

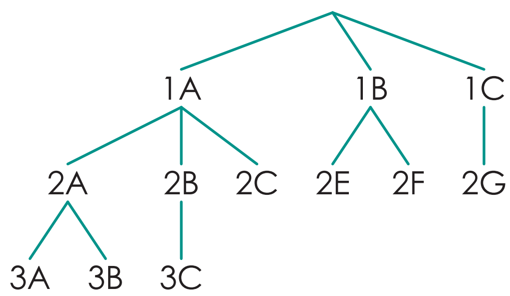
How many mathematically distinct first moves are there in Noughts and Crosses? (The correct answer is *three*—but what are they? We ignore the possibility of the first move being a X, or being a 0: it is the *location* of the first move that matters, here.)

By "mathematically distinct", I mean that if we have two different possibilities, but one of them can be transformed into the other by turning the board around, for example, or by some similar mathematical adjustment (such as a mirror reflection), we say that the two possibilities are mathematically one and the same. For example, placing a X in the top-left corner is mathematically identical to placing a cross in the bottom-right corner (or, in fact, in any corner).

How many mathematically distinct first moves are there in draughts (checkers), where *four* pieces begin in the front row?

How many mathematically distinct first moves are there in chess. (Remember that a Knight can jump over other pieces! Castling is *not* a first move; nor is *en passant*!)

To keep track of successive moves, as we analyse games, hunting for optimal strategies (and bad moves!), it helps to use a flow-chart that branches. If 1A, 1B, or 1C are the only possible first moves, then 2A, 2B, 2C, 2D, and 2E might be the only possible second moves (the replies by the second player), and 3A, 3B, 3C, and so on, might be the only possible third moves, etc.



Martin Gardner (1959, p. 42) notes that in Noughts and Crosses there are 15 120 possible moves ($9 \times 8 \times 7 \times 6 \times 5$) for the first five moves —not all mathematically distinct. However he says that "any astute youngster can become an unbeatable player with only an hour or so of analysis of the game" (Gardner, p. 42). Note that "unbeatable" is *not* the same as "always victorious"!

Philip Clarkson (2008) outlines some approaches to this analysis of Noughts and Crosses (or Tic-Tac-Toe, as he calls it, the common American name for the game). He also suggests varying the standard rules, and exploring the mathematical thinking, and strategy-solving of the resulting new variants. Of course, apart from the inherent mathematical interest of creating new problems to solve, the practical reason for changing the standard rules—for players who know the optimal strategy—is precisely that the fully analysed game has no further interest of its own. As I have argued (in my books of mathematics games), Noughts and Crosses is *not* a game for players who know the perfect drawing strategy, because they have no *choice* of how to move: they must follow the perfect strategy, or lose!

Clarkson's variants (3-D Stacking Cube Noughts and Crosses on a 3×3 board, and Bi-cube 3-D Noughts and crosses, using pieces made of two cubes, one White, one Black) are excellent. There are many other good variants to explore—next time.

From Helen Prochazka's

Scrapbook

Big Prime Hunters

A Mersenne prime is a special type of prime number: one that can be written in the form

$$2^p - 1.$$

Euclid discussed them in 350 BC but they bear the name of the 17th century French monk, Marin Mersenne, who made a study of them. The first few Mersenne primes are:

$$2^2-1,\ 2^3-1,\ 2^5-1,\ 2^7-1,\ \ldots$$

With a personal computer you have the chance to make maths history by discovering the next one Mersenne prime. The first step is the join Great Internet Mersenne Prime Search (GIMPS) project by going to www.mersenne.org. GIMPS was formed in 1996 and harnesses the power of hundreds of thousands of small home, school and business computers to search for more Mersenne primes.

The largest Mersenne prime, $2^{43\,112\,609} - 1$, was discovered in August 2008 by Edson Smith on a University of California computer. It is a mammoth number with 12 978 189 digits and would take about 3500 standard word processed pages if written out in full.

Why search for more primes? On a bigger scale there are some significant benefits of this mathematical research for our society. The GIMPS project has led to advances in distributed computing–that is using the Internet to effectively apply the unused computing power of thousands of machines. It has also led to advances in computer algorithms and flagged hidden hardware problems, helping to create better computer systems.

For individuals there is always the potential thrill of discovering something new and the mathematical fame would come with it.